Development of a Steady state solver for VOF methods

After successfully developing two and three-phase Volume of Fluid (VOF) methods such as THINC, CICSAM, STACS, HRIC, and MHRIC, we have embarked on enhancing our capabilities by creating a Steady-State accelerator and solver for VOF simulations. This report outlines the methodology behind this development.

Description of the method

To achieve a Steady-State using the VOF method already implemented in Phoenics we can consider the following approaches:

1. A full transient computation
2. A full transient computation with a false time step for the colour function
3. A full steady computation for all variables except the colour function solved with a false time step

In this report, we focus on the latter two methods (2 & 3) since their key modifications are centred on the treatment of the colour function (C) within the gxsurf.for subroutine, which plays a crucial role in VOF simulations. The governing equation for the colour function is as follow:

$$\frac{\partial C}{\partial t} + \nabla \cdot (VC) = 0$$

This PDE could be solved in conservative form or in non-conservative form depending on the physical problem.

For the first two methods 1) & 2) mentioned above, a limitation arises due to the explicit nature of the VOF method. The time step for the colour function equation is often more restrictive compared to the other equations within the simulation. This constraint can slow down the convergence towards steady-state.

Solution Enhancement

To accelerate the attainment of Steady State, we propose a solution involving the use of two separate time steps: one for the colour function equation and another for the remaining equations. This approach essentially introduces a "false" time step for the colour function. There are two options for this false time step:

1. **Fixed False Time Step for All Cells**: A uniform false time step is applied to all cells in the simulation domain.
2. **Spatially Variable False Time Step**: The false time step varies spatially, adapting to local conditions.

The method is *local-time stepping* (LTS), in which the time step is manipulated for each individual cell in the mesh, making it as high as possible to enable the simulation to reach steady-state quickly. This clearly violates the physics, described by the underlying equations of conservation of mass, momentum, *etc.* but the final solution is expected to satisfy the Physics of the problem.

Such an approach has been used in Open Foam and Fluent.

In both options, the false time step must satisfy the Courant-Friedrichs-Lewy (CFL) condition and account for surface tension restrictions. To distinguish between these choices, we introduce a logical variable (LTSS) in the gxsurf.for subroutine. When LTSS is false, the fixed false time step is employed, and when it is true, the spatially variable false time step is used.

To compute the Spatially Variable False time step, we compute first a local CFL and determine a False time step based on this CFL, then we determine a False time step based on sigma (surface tension) and a given false time step given by the user. The Spatially Variable False time step will be the minimum of these three false time steps.

In subroutine gxsurf.for this Spatially Variable False time step is stored in the field DFL. This field is then smoothed using the same smoothing procedure as the one introduced for surface tension but with a higher number of smoothing loop (>=5).

This smoothing is done to avoid abrupt changes in the Spatially Variable False time step, and then, it is used as the previous way of computing the colour function with a regular time step.

Using Full Steady Option

This is the third choice. A full steady computation for all variables except the colour function solved with a false time step.

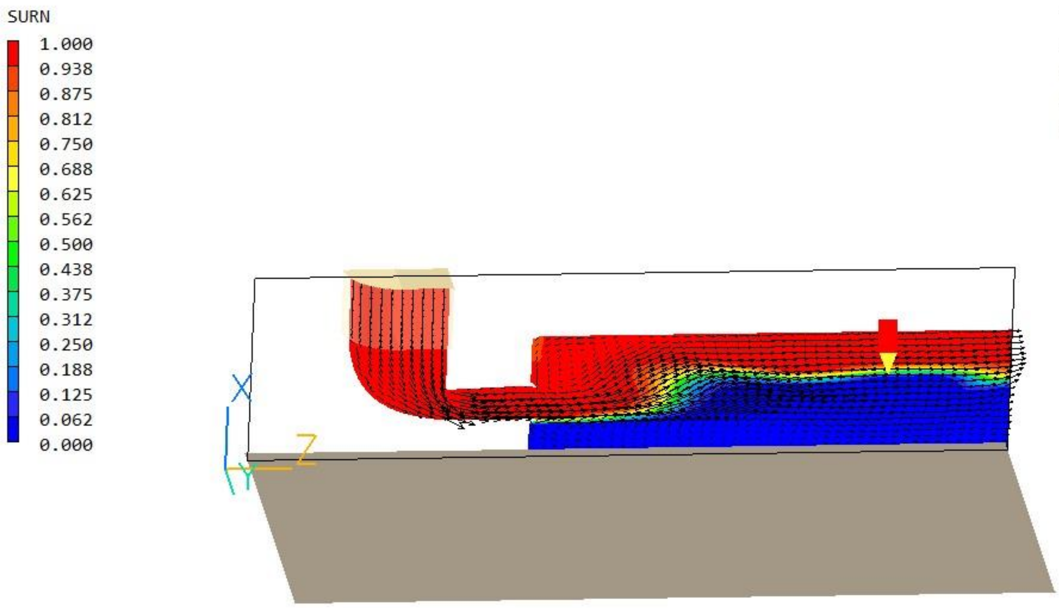
In this case, we choose STEADY option in the VR Menu but we introduce the possibility to activate VOF.

The VOF equation is then solved iteratively with a false time step. This iterative procedure is done for each sweep. A total number of iteration is chosen in advance actually 100 iteration with a test on the residual Max (Abs (C^{k+1}-C^{k}).

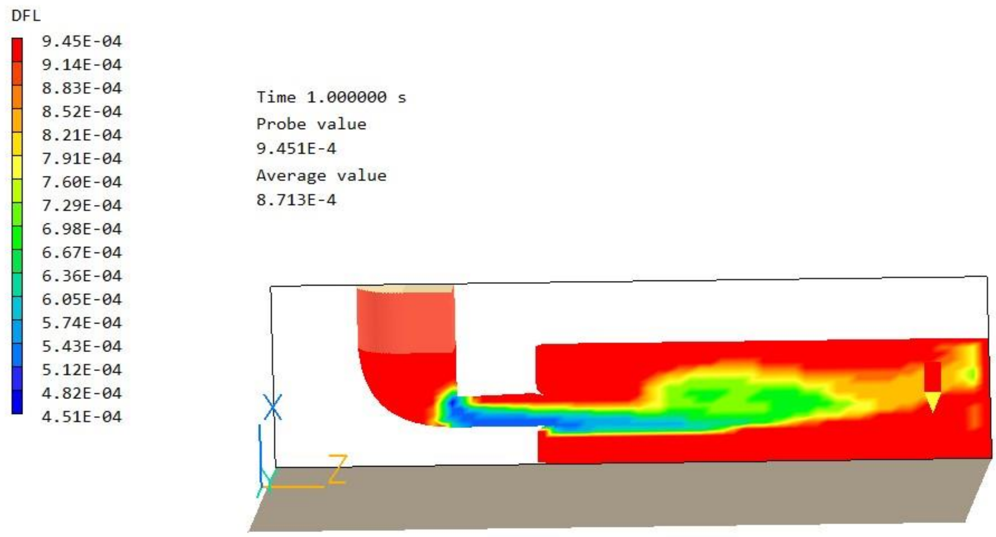
The choice of the iterative false time step follows the same principles as in the previous methods, whether it's a fixed false time step or a locally variable time step.

The GXSURF.for Subroutine contains all the changes made to achieve Steady State VOF.

Example of a 3D case done with this technique.



The time step for pressure and velocities is DT=1.e-03 whereas the time step for colour function is domain dependent as shown after.



Local time step for the colour function.

References:

1. Capodaglio, Giacomo, and Mark Petersen. 2022. “Local Time Stepping for the Shallow Water Equations in MPAS”. *Journal of Computational Physics* 449. Elsevier BV. doi:10.1016/j.jcp.2021.110818.
2. https://openfoam.org/release/2-0-0/steady-state-vof/
3. https://www.afs.enea.it/project/neptunius/docs/fluent/html/th/node298.htm